A Mathematical Model for Propagation of Freezing Interface in Lung Tumor during Cryosurgery

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ABSTRACT

Temperature distribution and position of freezing interface are determined in present mathematical model for cryosurgery of a tumor embedded in a lung by solving unsteady state bio-heat equation analytically together with boundary conditions. It is observed that first time depends upon the metabolic heat generation as well as on the heat due to local blood flow rate. The freezing interface velocity for both in healthy lung tissue and in tumor are calculated from the model and compared with available experimental results in the literature.

Keywords: Freezing Interface, Lung Tumor, Cryosurgery,

I. INTRODUCTION

Cryosurgery is an effective technique for the treatment of cancerous and non cancerous tumors that involves the use of very low temperature to destroy abnormal tissue. In order to apply cryosurgery more effectively, it is necessary to understand the propagation of freezing interface into the tissue. Most of the models discussed only one single type of tissue but, Comini and Giudice [5] considered freezing behavior during cryosurgery of Brain Tumor in contact with brain and bone tissue. Cooper and Trezek [4] predicted the lesion size for spherical and cylindrical cannulas in the steady state conditions. The rate of growth of the frozen region surrounding a cryoprobe was determined experimentally by embedding the cryoprobe in a clear gel by Cooper and Petrovie [2]. Bischof *et al*. [1] considered tumor /lung configuration for cryosurgery and solved the heat conduction equation under quasi-steady state conditions for the frozen region only. Some thermal properties of human blood during the freezing are described by Wessling and Blackshear [8]. Yang *et al*. [9] presented numerical method to simulate prostate cryosurgery by considering thermal stress aspects. Numerical techniques for planning of computerized cryosurgery were developed by Rossi *et al*. [7]. In the present paper the temperature distribution and position of freezing interface with time are obtained by solving bio-heat equations together with suitable boundary conditions to predict the exact position of frozen tissue of a tumor embedded in lung.

II. MATHEMATICAL MODELLING

The unsteady state bio-heat equation can be written as-

$$(\varrho c)_{ij} \frac{\partial T_{ij}}{\partial t} = K_{ij} \frac{\partial^2 T_{ij}}{\partial x^2} + S_m + S_b \qquad (1)$$

Where S_m and S_b denotes the frozen region the term e metabolic heat generation and heat due to local blood flow per unit volume. In the S_m and S_b vanish since the heat removed by blood (S_b) is equal to magnitude to the metabolic heat generation(S_m).

The macroscopic properties of lung computed by using void fractioning volumetric averaging techniques (Lunardini, [6]), and are given as follows

$$(\varrho C)_{\text{lung}} = (\varrho C)_w f_w + (\varrho C)_a f_a \qquad (2)$$

$$K_{\text{lung}} = K_w f_w + K_a f_a \qquad (3)$$

And

$$(\varrho)_{\text{lung}} = (\varrho)_w f_w + \varrho \ (C)_a f_a \qquad (4)$$

The void fractioning technique accounts for the insulation air pockets in the alveolus. These air pockets decrease the effective density specific heat, thermal conductivity and density. This effect can be neglected in the ratio for example thermal diffusivity etc. The problem is divided in two cases: in case I we consider only tumor or lung, and in case II the tumor imbedded in the healthy lung tissues.

Case I:

Again in this case there are two time domains to be considered the time $t_i^{(1)}$ before the surface temperature reaches the freezing value $T_{\rho h}$ after the insertion of the cryoprobe, during which no phase change occurs and the time $t_i^{(2)}$ when freezing interface reaches to a distance S and phase change occurs. The bio-heat equation and boundary conditions for the first time period $t_i^{(1)}$ in the affected tumor (or lung) regions are,

$$\frac{\partial T_{i2}}{\partial t}=\alpha_{i2}\frac{\partial^2 T_{i2}}{\partial x^2}-\frac{m_b C_b}{f_{i1}(\rho C)_{i2}}(T_{i2}-T_0), \qquad 0<x<S_i,$$
$$0<t<t_i^{(1)} \qquad (5)$$

$$T_{i2}(S_i,t)=T_0 \qquad (6)$$
$$\frac{\partial T_{i2}(S_i,t)}{\partial x}=0 \qquad (7)$$
$$\frac{\partial T_{i2}(0,t)}{\partial x}=\frac{T_{i2}(0,,t)-T_c}{R_{i2}} \qquad (8)$$
$$T_{i2}(x_i,\ t^{(1)}{}_1)=T_{ph} \qquad (9)$$
$$T_{i2}(x,0)=T_0 \qquad (10)$$
and $S_i(0)=0$ \qquad (11)

where i=1 for tumor and 2 for lung.

III. METHOD OF SOLUTION

Using heat balance integral technique in the affected region along with a quadratic temperature profile of the form
$$T_{i2}=a_i+b_ix+c_ix^2 \qquad (12)$$
And applying boundary conditions (6) to (11), we obtain

$$T_{i2}=T_0+\frac{(T_c-T_0)(x-\delta_i)^2}{\delta^2{}_i+2\delta_i R_{i2}} \qquad (13)$$

$$t_i^{(1)}{}_i=\frac{-1}{A_i}\left[\frac{4R^2{}_{i2}}{(B_i^2-4R_{i2}{}^2)}\quad \ln\left(1+\frac{\delta_i}{2R_{i2}{}^2}\right)+\right.$$
$$\left(\frac{B_i+4R_{i2}}{2(B_i+2R_{i2})}\right)\ln(1-\frac{\delta_i}{B_i})+$$
$$\left.\left(\frac{B_i+4R_{i2}}{2(B_i-2R_{i2})}\right)\ln(1+\frac{\delta_i}{B_i})\right] \qquad (14)$$

where
$A_i=\frac{m_b C_b}{f_{iw((\rho c))_{i2}}}$,

$B_i=\sqrt{\frac{6\alpha_{i2}}{A_i}}$ and $\frac{(T_0-T_{Ph})}{(T_{ph}-T_c)}=\frac{\delta_i}{2R_{i2}}$

The second time period begins with freezing as the first time period ends. On assuming constant heat flux at the inner boundary of interface i.e. the boundary inside unfrozen region, the heat conduction equation and the boundary conditions for the frozen region are

$$\frac{\partial T_{i1}}{\partial t}=\alpha_{i1}\frac{\partial^2 T_{i1}}{\partial x^2}, \quad 0<x<S, \quad t^{(1)}{}_i<t<\ t^{(2)}{}_i \quad (15)$$

$$T_{i1}(S_i,t)=T_{ph} \qquad (16)$$
$$\frac{\partial T_{i1}(0,t)}{\partial x}=\frac{T_{i1}(0,,t)-T_c}{R_{11}} \qquad (17)$$
$$S(0)=0 \qquad (18)$$
Where i=1 for tumor and 2 for lung.
The energy balance on freezing interface in tumor is given by the equation
$$H\rho_{11}\frac{dS}{dt}=K_{11}\frac{\partial T_{11}(S,t)}{\partial x}\text{-Q} \qquad (19)$$
and in lung
$$f_w\rho_{21}H\frac{dS}{dt}=K_{21}\frac{\partial T_{21}(S,t)}{\partial x}\text{-Q} \qquad (20)$$
Solving (15) using heat balance integral technique with a quadratic temperature profile
$$T_{i1}=T_{ph}+a_i(x-S)+b_i(x-S)^2 \qquad (21)$$
and applying the boundary conditions, we obtain
$$a_1=\frac{-2\alpha_{11}\rho_{11}H(R_{11}+S)+SQ(2R_{11}+S)}{2K_{11}S(2R_{11}+S)}+$$
$$\frac{[(2\alpha_{11}\rho_{11}H(R_{11}+S)-SQ(2R_{11}+S))^2+8\rho_{11}\alpha_{11}K_{11}SH(T_{ph}-T_c)(R_{11}+S)]^2}{2K_{11}S(2R_{11}+S)},$$

$$b_1=\frac{a_1Q-a^2{}_1K_{11}}{2\alpha_{11}\rho_{11}H},$$
$$a_2=\frac{-2\alpha_{21}\rho_{21}Hf_w(R_{21}+S)+SQ(2R_{21}+S)}{2K_{21}S(2R_{21}+S)}+$$
$$\frac{[(-2\alpha_{21}\rho_{21}f_wH(R_{21}+S)-SQ(2R_{21}+S))^2+8\rho_{21}\alpha_{21}K_{21}f_wH(T_{ph}-T_c)(2R_{21}+S)]^2}{2K_{21}S(2R_{21}+S)}$$

and $b_2=\frac{a_2Q-a^2{}_2K_{21}}{2\alpha_{21}\rho_{21}Hf_w}$

the second time period for tumor and lung are obtained by using (21) in (19) and (20) respectively and are given by

$$t^{(2)}{}_1=\int_0^S\frac{\rho_{11}H}{a_1K_{11}-Q}dS \qquad (22)$$
and
$$t^{(2)}{}_2=\int_0^S\frac{\rho_{21}Hf_w}{a_1K_{21}-Q}dS \qquad (23)$$

Case II:

In this case we consider the tumor embedded in healthy lung tissue. The temperature profile and time period for tumor in same as described in case first together with the condition that interface position S<L(length of tumor). But for the situation S≥L the statement of the problem is as follows

$$\frac{\partial T_{i1}}{\partial t}=\alpha_{i1}\frac{\partial^2 T_{i1}}{\partial x^2}, \quad 0<x<S_i, \quad S≥L \qquad (24)$$

Boundary conditions are

$$T_{21}(S,t)=T_{ph} \qquad (25)$$
$$T_{21}(L,t)=T_{11}(L,t) \qquad (26)$$
$$\frac{\partial T_{21}(L,t)}{\partial x}=K_{11}\frac{\partial T_{11}(L,t)}{\partial x} \qquad (27)$$

$$K_{11}\frac{\partial T_{11}(0,t)}{\partial x} = h[T_{11}(0,t) - T_c] \quad (28)$$

$$K_{21}\frac{\partial T_{21}(S,t)}{\partial x} = \rho_{21}Hf_w\frac{dS}{dt} + Q \quad (29)$$

Solving equation (24) along with the boundary conditions (25) to (29), we get

$$T_{11} = T_c + R_{11} + \frac{1}{K}[a_3 + 2b_3(L - S)] \quad (30)$$

$$T_2 = T_{ph} + a_3(x - S) + b_3(x - S)^2 \quad (31)$$

where

$$a_3 = \frac{-2\alpha_{21}\rho_{21}Hf_w(R_{11}+L+SK-LK)}{K_{21}(L-S)(2R_{11}+2L+SK-LK)} - \frac{Q}{2K_{21}} +$$
$$\frac{1}{2K_{21}}[\{\frac{-2\alpha_{21}\rho_{21}Hf_w(R_{11}+L+SK-LK)}{K_{21}(L-S)(2R_{11}+2L+SK-LK)}\}^2 -$$
$$\{\frac{8K_{21}\alpha_{21}\rho_{21}Hf_w(T_{ph}-T_c)}{(L-S)(2R_{11}+2L+SK-LK)}\}^{1/2}$$

$$b_3 = \frac{K(T_{ph}-T_c)-a_3(R_{11}+L+SK-LK)}{(L-S)(2R_{11}+2L+SK-LK)}$$

$$K = \frac{K_{11}}{K_{21}}$$

The interface velocity in the lung is given by

$$\frac{dS}{dt} = \frac{K_{21}a_3 - Q}{\rho_{21}Hf_w} \quad (32)$$

and time $t = \rho_{21}Hf_w\int_L^S \frac{dS}{K_{21}(T_c+R_{11})+\frac{1}{K}[a_3+2b_3(L-S)]}$

IV. RESULTS AND DISCUSSION

With the help of Matlab, graphical representation for the above analytical results are plotted from Fig. 1-2. The thermal constants used in the model are similar to Bischof *et al.* [1] and Comini and Giudice [5]. It is observed that freezing starts in tumor tissue after 70 seconds and after 4 seconds in healthy lung tissue respectively after the insertion of cryoprobe. While in some models, for example Bischof *et al.* [1] suggested that freezing starts as soon as the cryoprobe was inserted in the tissue. This difference is due to the reason that they did not consider first time period which is taken in this model. It is also observed that frozen time of a tumor tissue is much larger than that of the frozen time of a lung tissue of same length. Cooper and Trezek [3] had shown that when a cryoprobe with surface temperature minus 125 degree Celsius was embedded in brain tissue at an initial temperature of 37 degree Celsius, the total lesion size of 0.864 cm was formed in about 441 seconds.

Bischof *et al.* [1] predicted that a tumor of 1.0 cm long in lung was frozen in about 220 seconds. From figure 1 the position of freezing interface for tumors of length 1.0 cm embedded in the lung is shown.

It is observed that the frozen time for tumors are 452.48 seconds (for length 0.864 cm) and 515.64 seconds (for length 1.0 cm) respectively.
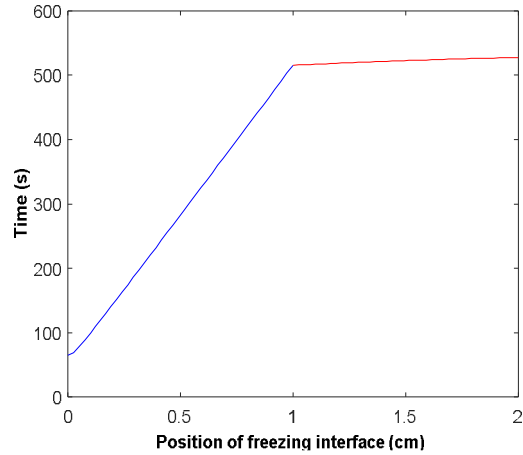
Fig. 1. Freezing interface in a tumor embedded in the lung, L=1.0 cm.
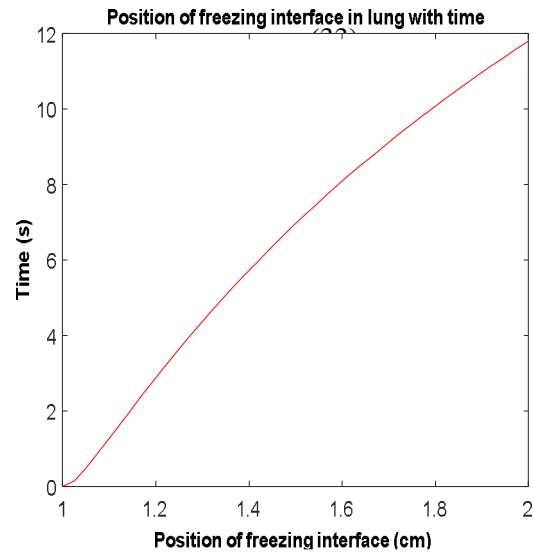
Fig. 2. Freezing interface in lung after crossing tumor lung boundary.

Thus it is clear that the results obtained from this model are very much close to the experimental results of Cooper and Petrovie [2]. It is clear from the figure 1 that the freezing interface moves very fast in the lung after crossing tumor lung boundary.

It can be seen from figure 1 that the frozen time for a lung tissue of size 1.0 cm is 11.6 seconds while in tumor of same size, the frozen time is 515.64 seconds.

[133]

V. CONCLUSION

It is observed that as the freezing interface crosses to tumor-lung boundary and enters in the lung, it moves very fast (Fig. 2). Therefore it is essential to save the healthy living tissues of lung around the tumor one has to consider the first time period and cryoprobe should be removed after the completion of second time period. The model presented in this paper can be used to predict the accurate position of freezing interface in a tumor embedded in the lung.

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